7 Trigonometry

M11CH7

7.1. ANGLE OF SLOPE AND GRADIENT

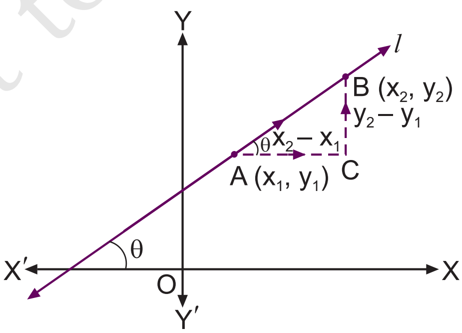
'Trigonometry' is the branch of Mathematics which deals with *angles*, whether of a triangle or not. 'Trigonometry' literally means, 'measurement of triangles'. The word, trigonometry is derived from the Greek words 'tri' (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure). So, originally 'Trigonometry' dealt with the study of relationships between the sides and angles of a triangle.

Angle of Slope and Gradient

Gradient of a line measures the steepness of a line. It is also called **slope** of the line and is usually denoted by '*m*'.

The gradient of a line is constant and is calculated by considering the coordinates of two points on the line.

Let a point move from one position $A(x_1, y_1)$ to another position $B(x_2, y_2)$ along a straight line l. The angle that line makes with the positive x-axis (the angle of inclination) is θ (See figure given below).



Then, the gradient or slope of the line

$$m = \frac{\text{Change in } y}{\text{Change in } x}$$

$$\text{Change in } y = \text{vertical change} = CB$$
$$= y_2 - y_1$$
$$\text{Change in } x = \text{horizontal change} = AC$$
$$= x_2 - x_1$$

Therefore, **the gradient of line** l

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x} \quad \ldots(1)$$

$$= \frac{\text{Difference in } y \text{ coordinate}}{\text{Difference in } x \text{ coordinates}}$$

Also, in ΔABC,

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\Rightarrow \quad \tan\theta = m \quad \ldots(2)$$

$$\Rightarrow \quad \theta = \tan^{-1}(m) \quad \ldots(3)$$

This is the **angle of slope** i.e., angle of inclination of line with positive x-axis.

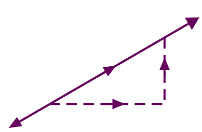
(i) A **horizontal line** neither rises nor falls. There is no change in y, because $y_1 = y_2$.

Gradient m = 0

(ii) For a **vertical line**, $x_1 = x_2$ so that there is no change in x. Since division by '0' is undefined, **gradient m is undefined**.

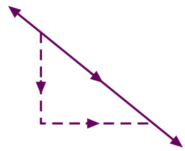
(iii) For lines rising from left to right, both changes are positive. As x increases, y also increases.

Gradient m is positive.



(iv) For lines falling from left to right, y decreases as x increases. Change in y is negative and change in x is positive.

Gradient m is negative.



Hence, the gradient of a line is a real number, positive, negative or zero.

Example 1: *Find the gradient of the line passing through the pair of points:*

(*i*) (2, – 4) and (5, 2)

(*ii*) (3, 5) and (4, 2)

Solution:

(i) Here $(x_1, y_1) = (2, -4)$

and $(x_2, y_2) = (5, 2)$

$\Rightarrow$ $x_1 = 2,\ y_1 = -4$ and

$x_2 = 5,\ y_2 = 2,\ x_1 \neq x_2.$

$$\therefore \quad \text{Gradient, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{5-2} = \frac{6}{3} = 2$$

The positive gradient indicates that the line is rising from left to right.

(ii) Here $(x_1, y_1) = (3, 5)$ and

$(x_2, y_2) = (4, 2)$

$\Rightarrow$ $x_1 = 3,\ y_1 = 5$ and

$x_2 = 4,\ y_2 = 2,\ x_1 \neq x_2$

$$\therefore \quad \text{Gradient, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{4-3} = \frac{-3}{1} = -3$$

The negative gradient indicates that the line is falling from left to right.

Example 2: *Find the angle of slope or gradient of the line passing through the pair of points:*

(*i*) (3, 5) and (– 2, 5)

(*ii*) (6, 3) and (6, – 7)

(*iii*) (2, 0) and (4, 4)

Solution:

(i) Here $(x_1, y_1) = (3, 5)$

and $(x_2, y_2) = (-2, 5)$

$\Rightarrow$ $x_1 = 3, y_1 = 5$

and $x_2 = -2,$

$y_2 = 5, x_1 \neq x_2$

$\therefore$ Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-5}{-2-3} = \dfrac{0}{-5} = 0$

so that the line is horizontal.

$\therefore$ Angle of gradient,

$$\begin{aligned} \theta &= \tan^{-1} m \\ &= \tan^{-1} 0 = 0° \end{aligned}$$

(ii) Here $(x_1, y_1) = (6, 3)$

and $(x_2, y_2) = (6, -7)$

$\Rightarrow$ $x_1 = 6, y_1 = 3$ and

$x_2 = 6, y_2 = -7$

Since $x_1 = x_2$, gradient m is undefined.

Also, angle of gradient is undefined

(iii) Here $(x_1, y_1) = (2, 0)$

and $(x_2, y_2) = (4, 4)$

$\Rightarrow$ $x_1 = 2, y_1 = 0$ and $x_2 = 4, y_2 = 4$

$\Rightarrow$ Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-2}{4-0} = \dfrac{2}{4} = \dfrac{1}{2} = 0.5$

Now, angle of slope, $\theta = \tan^{-1} (m)$

$$\begin{aligned} &= \tan^{-1} (0.5) \\ &= 26.57° \end{aligned}$$

EXERCISE 7.1

1. Find the gradient of the lines passing through the points:

(i) $(-3, -2)$ and $(1, 4)$.

(ii) $(4, -7)$ and $(8, -7)$.

(iii) $(5, -4)$ and $(5, 6)$.

(iv) $(0, -3)$ and $(-6, 3)$.

2. Find the angle of slopes or gradient of the line which passes through the points:

(i) (1, 1) and (7, 2)

(ii) (0, 0) and $(\sqrt{3}, 1)$

(iii) (0, –5) and (4, 5)

(iv) (1, 5) and (4, 11)

(v) (– 1, – 4) and (– 6, – 16)

7.2. SINE, COSINE AND TANGENT OF ACUTE ANGLES

Trigonometric Ratios

There are three basic trigonometric ratios–**sin** (sine), **cos** (cosine) and **tan** (tangent). Each of these relates an angle of a right angled triangle to a ratio of the lengths of two of its side. The sides of right angled triangle have specific names, two of which are dependent on their position in relation to a specific angle.

Consider a right angled triangle ABC, right angled at B. ∠BAC and ∠BCA are acute angles. They are briefly written as ∠A and ∠C respectively. AC is the **hypotenuse** of the right triangle *i.e.*, the longest side of the right angled triangle which is always opposite the right angle.

With respect to angle A, the side BC is called the opposite side and side AB is called the adjacent side (see Fig. i).

With respect to angle C, the side AB is called the opposite side and BC is called the adjacent side (See Fig. ii).

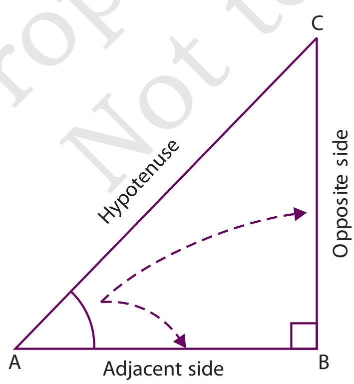


Fig. (i)

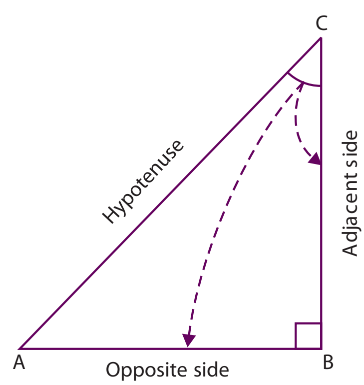


Fig. (ii)

For an acute angle of a right angled triangle, label the sides as follows:

- the side opposite to the right angle is called the **hypotenuse**,

- the side facing an acute angle under consideration is called the **opposite side** to that angle and
- the third side, which is a part of the acute angle under consideration, is called the **adjacent side** to that angle.

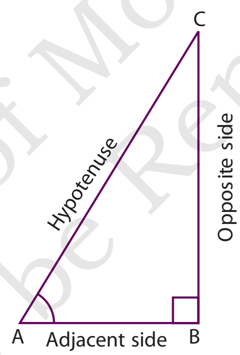
Now, let us define three basic trigonometric ratios involving the sides of a right angled triangle. These ratios are called **trigonometric ratios**. The trigonometric ratios of angle A are defined as:

1. Sine of an Angle

Sine of an angle A is given by:

$$\text{sine of } \angle A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

Sine of $\angle A$ is briefly written as sin A (Taking the first three letters of the word sine).



Thus, $$\sin A = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

2. Cosine of an Angle

Cosine of an angle A is given by:

$$\text{Cosine of } \angle A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

Cosine of $\angle A$ is briefly written as cos A (Taking the first three letters of the word cosine).

Thus, $$\cos A = \frac{\text{Adjacent side}}{\text{Hypotenous}} = \frac{AB}{AC}$$

3. Tangent of an Angle

Tangent of an angle A is given by:

$$\text{tangent of } \angle A = \frac{\text{Side apposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{BC}{AB}$$

Tangent of $\angle A$ is briefly written as tan A (taking the first three letters of the word tangent)

Thus, $$\tan A = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB}$$

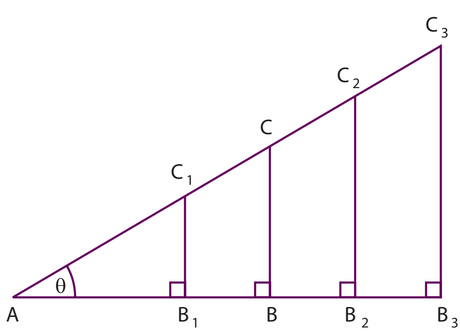
Clearly, $$\tan A = \frac{BC}{AB} = \frac{\frac{BC}{AC}}{\frac{AB}{AC}} = \frac{\sin A}{\cos A}$$

In the right angled triangle ABC, right angled at B, let us take points C_1, C_2, C_3 on the hypotenuse (or the hypotenuse produced) and draw C_1B_1, C_2B_2, C_3B_3, perpendiculars on AB. Clearly, triangles ABC, AB_1C_1, AB_2C_2 and AB_3C_3 are similar. Therefore, the corresponding sides of these triangles are proportional and we have

$$\sin\theta = \frac{AC}{BC} = \frac{B_1C_1}{AC_1} = \frac{B_2C_2}{AC_2} = \frac{B_3C_3}{AC_3}$$

$$\cos\theta = \frac{AB}{AC} = \frac{AB_1}{AC_1} = \frac{AB_2}{AC_2} = \frac{AB_3}{AC_3}$$

$$\tan\theta = \frac{BC}{AB} = \frac{B_1C_1}{AB_1} = \frac{B_2C_2}{AB_2} = \frac{B_3C_3}{AB_3}$$

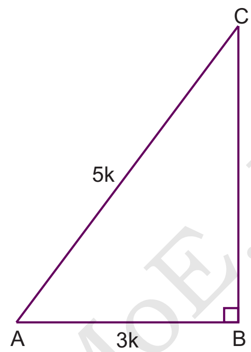


Thus, the values of trigonometric ratios are independent of the lengths of sides of the triangle.

For example, $\cos A = \frac{3}{5} \Rightarrow \frac{AB}{AC} = \frac{3}{5}$

$\Rightarrow AB : AC = 3 : 5$

If we take $AB = 3k$, then $AC = 5k$, where k is an arbitrary positive number.



Using Pythagoras theorem, we can find the third side BC.

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad (3k)^2 + BC^2 = (5k)^2$$

$$\Rightarrow \quad BC^2 = 25k^2 - 9k^2 = 16k^2 = (4k)^2$$

$$\Rightarrow \quad BC = 4k$$

Now, we can write the values of sin A and tan A as:

$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

and

$$\tan A = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$$

Thus, if any of the three trigonometric ratios is known, then the ratio of two sides of the right triangle is known. Using Pythagoras theorem, we can find the third side and hence the values of other two trigonometric ratios.

Note 1: $\sin A \neq \sin \times A$, $\cos A \neq \cos \times A$, $\tan A \neq \tan \times A$

Note 2: If $\sin \theta = \frac{1}{2}$, then θ is the angle whose sine is $\frac{1}{2}$. We write it as: $\theta = \sin^{-1}\left(\frac{1}{2}\right)$ and read it as θ is the sine inverse of $\frac{1}{2}$

Similarly,

$$\cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$= \text{cosine inverse of } \frac{2}{3}$$

$$= \text{the angle whose cosine is } \frac{2}{3}$$

$$\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1)$$

$$= \text{tangent inverse of } 1$$

$$= \text{the angle whose tangent is } 1.$$

How to Use Trigonometric Tables/Calculator, to Find the Trigonometric Ratios

Let us use the following example to explain how to use trigonometric tables /calculator to find sines, cosines and tangents of acute angles.

[**Note:** If the trigonometric tables are not specifically asked for, the calculator or the tables can be used to solve the given problem.]

Example 3: *Read the values of the following from tables and calculators:*

(*i*) *sin 42°* (*ii*) *cos 77°* (*iii*) *tan 65°*

Solution: Using Tables

(i) From the table of sines

Degrees

⋮ sin 42° = 0.6691

42 → 0.6691

(ii) From the table of cosines

Degrees

⋮ cos 77° = 0.2250

77 → 0.2250

(iii) From the table of tangents

Degrees

⋮ tan 65° = 2.1445

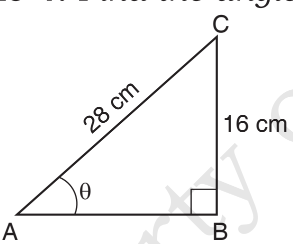
65 → 2.1445

Using Calculator

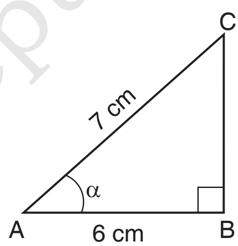
(i)	Display	42	0.669130606 $\approx$ 0.6691
	Keystroke	$\boxed{\text{sin}}$	(correct to four decimal places)
(ii)	Display	77	0.224951054 $\approx$ 0.2250
	Keystroke	$\boxed{\text{cos}}$	(correct to four decimal places)
(iii)	Display	65	2.144506921 $\approx$ 2.1445
	Keystroke	$\boxed{\text{tan}}$	(correct to four decimal places)

Example 4: *Find the angles marked in the following figures:*

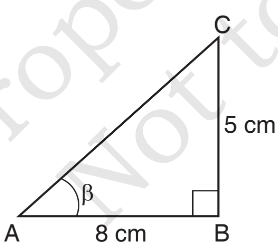
(*i*)



(*ii*)



(*iii*)



Solution:

(i) From the figure, sides with respect to angle θ are:

Opposite side, BC = 16 cm

Hypotenuse, AC = 28 cm

$$\therefore \qquad \sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\Rightarrow \qquad \sin\theta = \frac{16}{28} = \frac{4}{7}$$

$$\Rightarrow \qquad \theta = 34.8° \qquad \left(\because \theta = \sin^{-1}\frac{4}{7}\right)$$

(ii) From the figure, sides with respect to angle α are:
Adjacent side, AB = 6 cm
Hypotenuse, AC = 7 cm

$$\therefore \qquad \cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\Rightarrow \qquad \cos\theta = \frac{6}{7}$$

$$\Rightarrow \qquad \theta = 31° \qquad \left(\because \theta = \cos^{-1}\frac{6}{7}\right)$$

(iii) From the figure, sides with respect to angle β are:
Opposite side, BC = 5 cm
Adjacent side, AB = 8 cm

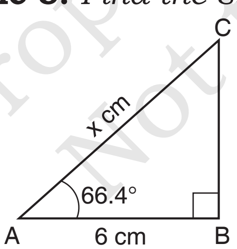
$$\therefore \qquad \tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB}$$

$$\Rightarrow \qquad \tan\theta = \frac{5}{8}$$

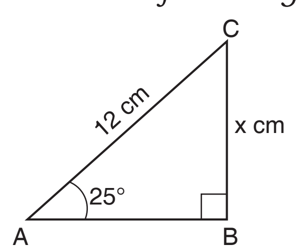
$$\Rightarrow \qquad \theta = 32° \qquad \left(\because \theta = \tan^{-1}\frac{5}{8}\right)$$

Example 5: *Find the side marked with letters in the following figures.*

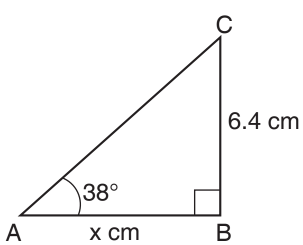
(*i*)



(*ii*)



(*iii*)



Solution:

(i) From the figure, sides with respect to given angle 66.4° are:

Adjacent side, AB = 6 cm

Hypotenuse, AC = x cm

Here, we know adjacent side to 66.4° and want to find the hypotenuse (AC). So we use cosine formula

$$\therefore \quad \cos 66.4^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \cos 66.4^\circ = \frac{6}{x}$$

$$\Rightarrow \quad x = \frac{6}{\cos 66.4^\circ}$$

$$= \frac{6}{0.4} = 15 \text{ cm}$$

(ii) From the figure, we know the hypotenuse (AC) and want to find opposite side (BC). Therefore, we use the sine formula

$$\therefore \quad \sin 25^\circ = \frac{\text{Opposite side (BC)}}{\text{Hypotenuse (AC)}}$$

$$\Rightarrow \quad \sin 25^\circ = \frac{x}{12}$$

$$\Rightarrow \quad x = 12 \times \sin 25^\circ$$

$$= 12 \times 0.4226$$

$$= 5.07 \text{ cm (to 2 decimal point)}$$

(iii) From the figure, we know opposite to side (BC) and want to find adjacent side (AB). Therefore, we use tangent formula.

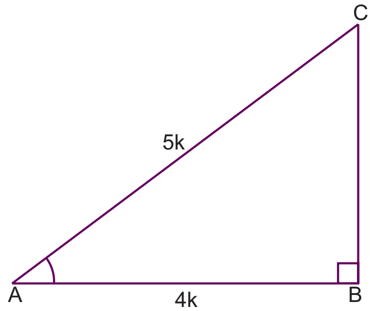
$$\therefore \quad \tan 38^\circ = \frac{\text{Opposite side (BC)}}{\text{Adjacent side (AB)}}$$

$$\Rightarrow \quad \tan 38^\circ = \frac{6.4}{x}$$

$$\Rightarrow \quad x = \frac{6.4}{\tan 38^\circ} = \frac{6.4}{0.7813} = 8.16 \text{ cm}$$

Example 6: *Given* $\cos A = \frac{4}{5}$, *find sin A and tan A.*

Solution: Draw a right angled triangle ABC, right angled at *B*.



Given: $\cos A = \frac{4}{5}$

$\Rightarrow \quad \frac{AB}{AC} = \frac{4}{5}$

Take AB = $4k$, then AC = $5k$, k being a positive number.

By Pythagoras theorem, $AB^2 + BC^2 = AC^2$

$$\Rightarrow \quad (4k)^2 + BC^2 = (5k)^2$$

$$\Rightarrow \quad BC^2 = 25k^2 - 16k^2$$

$$= 9k^2 = (3k)^2$$

$$\Rightarrow \quad BC = 3k$$

By definition, $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{4}$

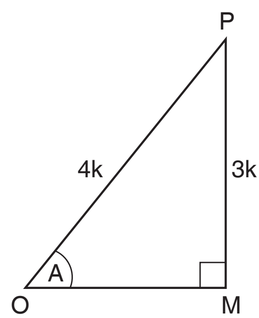
$$\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

Example 7: *If $\sin A = \frac{3}{4}$, calculate cos A and tan A.*

Solution: Let us first draw a right angled triangle OMP, right angled at B.

Using the definition of sine, we have

$$\sin A = \frac{MP}{OP} = \frac{3}{4} \quad \Rightarrow \quad \frac{MP}{OP} = \frac{3k}{4k}$$



Take MP = $3k$, then OP = $4k$, k being a positive number.

Now, by using the Pythagoras Theorem, we have

$$OP^2 = OM^2 + MP^2$$

$$\Rightarrow \quad (4k)^2 = OM^2 + (3k)^2$$

$$\Rightarrow \quad OM^2 = (4k)^2 - (3k)^2 = 16k^2 - 9k^2 = 7k^2$$

$$\Rightarrow \quad OM^2 = (\sqrt{7}\,k)^2$$

$$\Rightarrow \quad OM = \sqrt{7}\,k$$

$\therefore$ By definition,

(i) $\cos A = \dfrac{OM}{OP} = \dfrac{\sqrt{7}\,k}{4k} = \dfrac{\sqrt{7}}{4}$

(ii) $\tan A = \dfrac{MP}{OM} = \dfrac{3K}{\sqrt{7}K} = \dfrac{3}{\sqrt{7}}$

EXERCISE 7.2

1. Find, without using tables, the values of $\cos\theta$ and $\tan\theta$, if $\sin\theta = \dfrac{5}{13}$.

2. If $\cos p = \dfrac{24}{25}$ where $0° < p < 90°$, find $\sin p$.

3. If $\cos p = \dfrac{3}{5}$ where $0° < p < 90°$, find the value of $\dfrac{\tan p}{\cos p}$.

4. If $0 \le \theta \le 90°$ and $\tan\theta = \dfrac{3}{4}$, find $\sin\theta$ and $\cos\theta$.

5. If $\sin x = \dfrac{1}{q}$, find $\tan x$ and $\cos x$.

7.3. INVERSE OF TRIGONOMETRIC RATIOS

If $\sin\theta = x$, then θ is the angle whose sine is x. We write it as $\theta = \sin^{-1}x$ and read it as 'θ is the sine inverse of x'.

For example, $\sin 30° = \dfrac{1}{2}$

$$\Rightarrow \quad \sin^{-1}\frac{1}{2} = \text{the angle whose sine is } \frac{1}{2}$$
$$= 30°$$

If $\cos\theta = x$, then θ is the angle whose cosine is x. We write it as $\theta = \cos^{-1} x$ and read it as 'θ is the cosine inverse of x'.

For example, $\cos 60° = \frac{1}{2}$

$$\Rightarrow \qquad \cos^{-1}\frac{1}{2} = \text{the angle whose cosine is } \frac{1}{2} = 60°$$

If $\tan\theta = x$, then θ is the angle whose tangent is x. We write it as $\theta = \tan^{-1} x$ and read it as 'θ is the tangent inverse of x'.

For example, $\tan 45° = 1$

$$\Rightarrow \qquad \tan^{-1} 1 = \text{the angle whose tangent is } 1$$
$$= 45°$$

The important point to be borne in mind is that every inverse trigonometric ratio is an angle which can be determined by using tables or calculators.

Example 8: *Read the following angles using tables and calculators:*

(*i*) $\sin^{-1}(0.3907)$

(*ii*) $\cos^{-1}(0.4848)$

(*iii*) $\tan^{-1}(0.7536)$

Solution: Using Tables

Run your eye through the column next to degrees.

(i) From the table of sines

Degrees

⋮ $\qquad \sin^{-1}(0.3907)$ = the angle whose sine is 0.3907

23 ← 0.3907 = 23°

(ii) From the table of cosines

Degrees

⋮ $\qquad \cos^{-1}(0.4848)$ = the angle whose cosine is 0.4848

61 ← 0.4848 = 61°

(iii) From the table of tangents

Degrees

⋮ $\qquad \tan^{-1}(0.7536)$ = the angle whose tangent is 0.7536

37 ← 0.7536 = 37°

Using Calculator

Step I. Display the number against the inverse trigonometric ratio.

Step II. Press the key marked $\boxed{\text{2nd F}}$.

Step III. Press the key marked the 'corresponding trigonometric ratio'. Thus, press the key marked.

sin	to find	$\sin^{-1}$
cos	to find	$\cos^{-1}$
tan	to find	$\tan^{-1}$

Step II **Step I**

(i) $\boxed{\text{2nd F}}$ 0.3907

$\sin^{-1}$

Step III $\boxed{\sin}$

$$\sin^{-1}(0.3907) = 22.99806246 \text{ degrees}$$
$$= 23° \text{ (rounded off to nearest degree)}$$

Step II **Step I**

(iii) $\boxed{\text{2nd F}}$ 0.4848

$\cos^{-1}$

Step III $\boxed{\cos}$

$$\cos^{-1}(0.4848) = 61.00063021 \text{ degrees}$$
$$= 61° \text{ (rounded off to nearest degree)}$$

Step II **Step I**

(iii) $\boxed{\text{2nd F}}$ 0.7536

$\tan^{-1}$

Step III $\boxed{\tan}$

$$\tan^{-1}(0.7536) = 37.00167917 \text{ degrees}$$
$$= 37° \text{ (rounded off to nearest degree)}$$

EXERCISE 7.3

Read the following angles using tables and calculators:

1. $\sin^{-1}(0.9205)$

2. $\sin^{-1}(0.1219)$

3. $\sin^{-1}(0.7314)$

4. $\cos^{-1}(0.9744)$

5. $\cos^{-1}(0.1045)$

6. $\cos^{-1}(0.7771)$

7. $\tan^{-1}(0.1944)$

8. $\tan^{-1}(1.2799)$

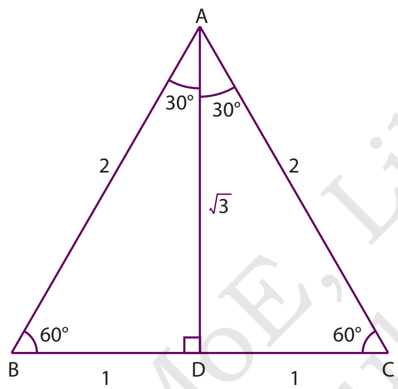
9. $\tan^{-1}(3.0777)$.

7.4. THE TRIGONOMETRIC RATIOS OF 30°, 45° AND 60°

Trigonometric Ratios of 30° and 60°

Draw an equilateral triangle ABC with each side 2 units. Since each angle in an equilateral triangle is 60° *i.e.*, $\angle A = \angle B = \angle C = 60°$.

Draw AD perpendicular on BC. Then triangles ADB and ADC are congruent so that BD = DC = 1 unit.



In right triangle ADB, by Pythagoras theorem,

$$AD^2 + BD^2 = AB^2$$

$$\Rightarrow \quad AD^2 + 1^2 = 2^2$$

$$\Rightarrow \quad AD^2 = 4 - 1 = 3$$

$$\Rightarrow \quad AD = \sqrt{3} \text{ units}$$

Now in right triangle ADB, with respect to ∠BAD = 30°

Opposite side = BD = 1 unit,

Adjacent side = AD = $\sqrt{3}$ units,

Hypotenuse = AB = 2 units.

Therefore,

$$\sin 30° = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BD}{AB} = \frac{1}{2}$$

$$\cos 30° = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BD}{AD} = \frac{1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}}$$

Also in right triangle ADB, with respect to $\angle$ABD = 60°

Opposite side = AD = $\sqrt{3}$ units,

Adjacent side = BD = 1 unit

Hypotenuse = AB = 2 units.

Therefore,

$$\sin 60° = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BD}{AB} = \frac{1}{2}$$

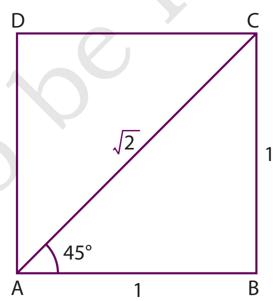
$$\tan 60° = \frac{\text{Opposite side}}{\text{Adjacents side}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Trigonometric Ratio of 45°

Draw a square ABCD of side 1 unit. Draw the diagonal AC.

In right angled triangle ABC, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$= 1^2 + 1^2 = 2$$
$$\Rightarrow \quad AC = \sqrt{2} \text{ units}$$



Also, triangles ABC and ADC being congruent, $\angle$BAC = $\angle$DAC = 45°.

In right triangle ABC with respect to $\angle$BAC = 45°.

Opposite side = BC = 1 unit, adjacent side = AB = 1 unit hypotenuse = AC = $\sqrt{2}$ units

Therefore,

$$\sin 45° = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB} = \frac{1}{1} = 1.$$

The table below gives the values of three basic trigonometric ratio of 0°, 30°, 45°, 60° and 90°.

∠A	**0°**	**30°**	**45°**	**60°**	**90°**
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

Remark: From the table above you can observe that as ∠A increases from 0° to 90°, sin A increases from 0 to 1 and cos A decreases from 1 to 0.

Example 9: *Evaluate:* $\dfrac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ}$

Solution: Given expression $= \dfrac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ} = \dfrac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}} = \dfrac{\sqrt{3}}{2}$

Example 10: *Evaluate the following:*

(*i*) *sin 60° cos 30° + sin 30° cos 60°*

(*ii*) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$.

Solution: (i) Here, we have

sin 60° cos 30° + sin 30° cos 60°

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1.$$

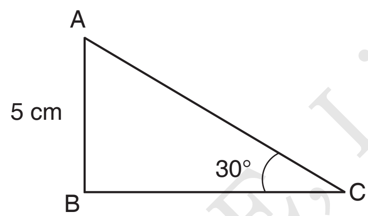
Hence, the value of the given expression is 1.

(ii) We have,

$2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = 2$$

Example 11: *In ΔABC, right-angled at B, AB = 5 cm and $\angle ACB = 30°$ (as shown in figure). Determine the lengths of the sides BC and AC.*



Solution: To find the length of the side BC, we will choose the trigonometric ratio involving BC and the given side AB. Since BC is the side adjacent to angle C and AB is the side opposite to angle C, therefore

$$\frac{AB}{BC} = \tan C$$

$$\Rightarrow \quad \frac{5}{BC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BC = 5\sqrt{3} \text{ cm}$$

To find the length of the side AC, we consider

$$\sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{5}{AC}$$

$$\Rightarrow \quad AC = 10 \text{ cm}$$

Note that alternatively we could have used Phythagoras theorem to determine the third side in the example above.

i.e., $$AC = \sqrt{AB^2 + BC^2} = \sqrt{5^2 + (5\sqrt{3})^2} \text{ cm}$$

$$= 10 \text{ cm}$$

Example 12: *Find the value of θ ($0° \le \theta \le 90°$) satisfying the equations.*

(i) $\sin\theta = \frac{1}{2}$ (ii) $\cos\theta = \frac{1}{\sqrt{2}}$ (iii) $\tan\theta = \sqrt{3}$

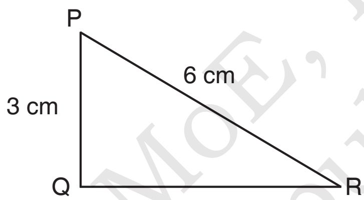
Solution:

(i) $\sin\theta = \frac{1}{2} \Rightarrow \sin\theta = \sin 30° \Rightarrow \theta = 30°$

(ii) $\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \cos 45° \Rightarrow \theta = 45°$

(iii) $\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60° \Rightarrow \theta = 60°$

Example 13: *In ΔPQR, right-angled at Q (as shown in figure), PQ = 3 cm and PR = 6 cm. Determine ∠QPR and ∠PRQ*



Solution: Given: PQ = 3 cm and PR = 6 cm.

Therefore, $\frac{PQ}{PR} = \sin R$

$\Rightarrow \sin R = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \sin R = \sin 30°$

So, $R = \angle PRQ = 30°$

Now, in a right-angled triangle PQR,

sum of all angles = 180°

$\Rightarrow \angle QPR + \angle PRQ + \angle RQP = 180°$

$\Rightarrow \angle QPR + 30° + 90° = 180°$

$\Rightarrow \angle QPR = 180° - 120° = 60°$

EXERCISE 7.4

1. Evaluate the following expression:

(i) $\sin 60° \cos 30° - \sin 30° \cos 60°$

(ii) $(\cos 30°)^2 + 2(\tan 45°)^2 - (\sin 60°)^2$

(iii) $\dfrac{\sin 30° \cos 45°}{\tan 60°}$

(iv) $\dfrac{\sin 30° + \tan 45°}{\cos 30° - \tan 60°}$

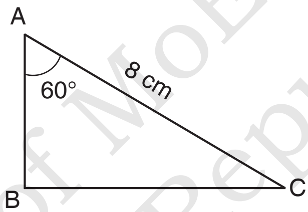
2. Find the values of θ (0° ≤ θ ≤ 90°) satisfying the following equations.

(i) $\cos\theta = 1$ (ii) $\sin\theta = 1$

(iii) $\tan\theta = 1$ (iv) $\sin\theta = \dfrac{\sqrt{3}}{2}$

(v) $\cos\theta = \dfrac{1}{2}$ (vi) $\tan\theta = \dfrac{1}{\sqrt{3}}$

3. In the figure, ΔABC is a right angled at B, AC = 8 cm and ∠BAC = 60°. Find the lengths of the sides AB and BC.

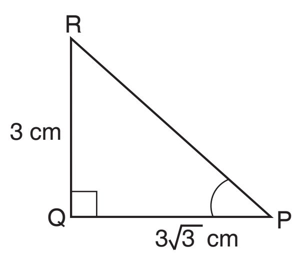


4. In ΔPQR, right angled at Q (as shown in figure), PQ = $3\sqrt{3}$ cm and RQ = 3 cm. Find

(i) ∠P or ∠RPQ

(ii) ∠R or ∠QRP

(iii) length of the side PR.

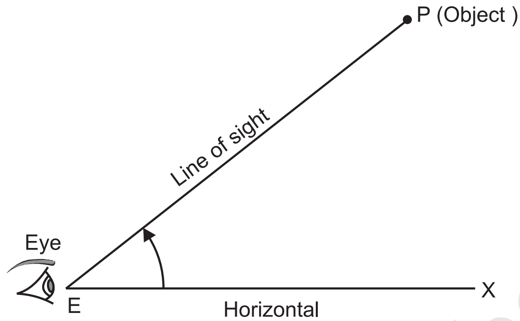


7.5. ANGLES OF ELEVATION AND DEPRESSION

Trigonometry helps in finding the **heights** of objects and the **distances** between points where actual measurements are very difficult.

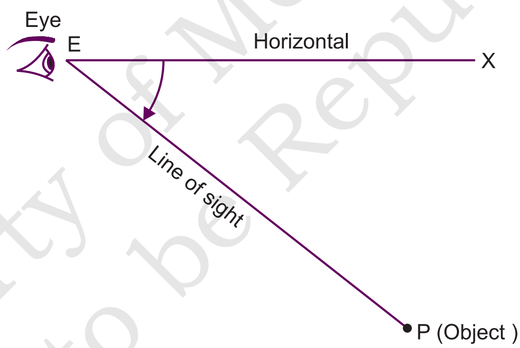
For example, trigonometry enables us to find the heights of mountains, breadths of rivers, distances from earth to the planets and stars etc.

Let E be the eye of an observer and EX, the horizontal level through E. Let P be an object. The line EP is called the **line of sight**.

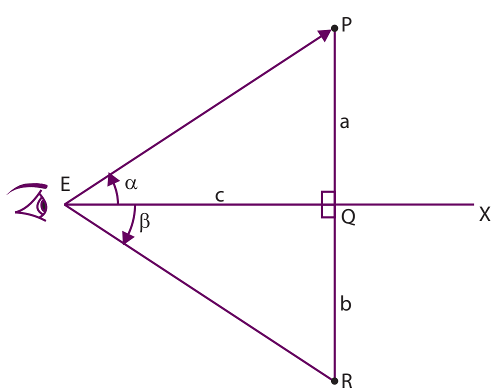


If P is above EX, as shown in figure below, then ∠XEP is called the angle of elevation of P as viewed from E. In this case, we have to raise our head to look at P.

If P is below EX, as shown in figure below, then ∠XEP is called the **angle of depression** of P as viewed from E. In this case, we have to lower our head to look at P.



Thus, **angle of elevation** of a point P is the angle between the line sight EP and the horizontal EX when the point P is above the horizontal (see figure below).



If PQ $\perp$ EX and PQ = a, EQ = c, then the angle of elevation α is given by :

$$\tan \alpha = \frac{a}{c}$$

$$\Rightarrow \qquad \alpha = \tan^{-1}\left(\frac{a}{c}\right)$$

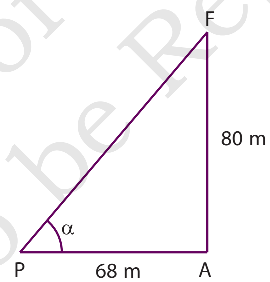
Angle of depression of a point R is the angle between the line of sight ER and the horizontal EX when the point R is below the horizontal (see figure above).

If RQ $\perp$ EX and RQ = b, EQ = c, then the angle of depression β is given by:

$$\tan \beta = \frac{b}{c} \quad \Rightarrow \quad \beta = \tan^{-1}\left(\frac{b}{c}\right)$$

Example 14: *What is the angle of elevation of a vertical flagstaff of height 80 m from a point 68 m from its fort?*

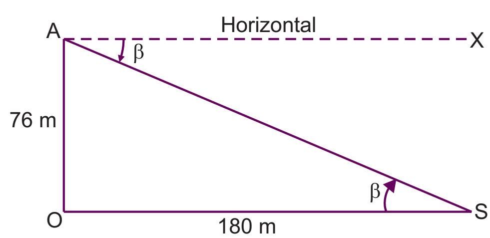
Solution: Let AF = 80 m be the flagstaff and P, the point on the ground such that AP = 68 m. The angle of elevation of F as observed from P is APF. If $\angle$APF = α, then



$$\tan \alpha = \frac{AF}{AP} = \frac{80}{68} = 1.1765$$

$$\Rightarrow \qquad \alpha = \tan^{-1}(1.1765) = 49.6°$$

Example 15: *The top of a vertical cliff is 76 m above the sea level. A ship is 180 m from the fort of the cliff. Find the angle of depression of the ship as viewed from the top of the cliff.*



Solution: Let OA = 76 m be the vertical cliff and S, the ship such that OS = 180 m. If AX is the horizontal through A, then angle of depression of S as observed from A is XAS.

Let $\angle XAS = \beta$, then $\angle OSA = \beta$ (Alternate angles)

and $$\tan \beta = \frac{76}{180} = 0.4222$$

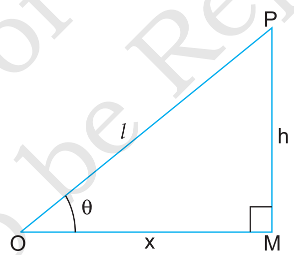
$$\Rightarrow \quad \beta = \tan^{-1} (0.4222) = 22.9°.$$

7.6. HEIGHT AND DISTANCE

Relations between Angles and Sides of A Right-Angled Triangle

While studying trigonometric ratios of an angle, we have learnt the concept of a **fundamental triangle** OMP (which is a right angled triangle).

In right ΔOMP, we shall denote the point of observation (*i.e.*, the eye of an observer) by O, object by P, the height of the object P above the horizontal level by h, the line of vision OP by l, the angle of elevation/depression of P by θ. Thus, we shall be dealing with four variables namely x, h, l and θ. Out of which, at least two are given in a problem.



Then, in rt. ΔOMP, we have

$$\sin \theta = \frac{h}{l}, \cos \theta = \frac{x}{l}, \tan \theta = \frac{h}{x}.$$

Using the above notations in problems of heights and distances, the unknown variable may be obtained by using the relation given below.

$$\frac{\text{Required side}}{\text{Known/Give side}} = \text{T-Ratio of the given angle.}$$

3-Step Working Rule for Solving Problems of Heights and Distances

While solving problems on heights and distances, we can employ a general method involving the following **3-Step Working Rule**:

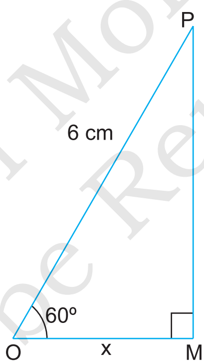
Step 1 *First of all, draw the diagrammatic representation of the given problems in the form of the fundamental* ΔOMP *with usual notations (i.e., l, h, x and θ).*

Step 2 *Next, identify the required side and the given data. Use the following relation:*

$$\frac{\textit{Required side}}{\textit{Known/ Given side}} = \text{T-Ratio of the given angle.}$$

Step 3 *Using the values of T-ratios of standard angles (i.e., 30°, 45°, and 60°), proceed to simplify the above result to get the required side.*

Example 16: *In the figure, find x.*



Solution: Here, we have OP, l = 6 cm, $\angle MOP$, θ = 60° and OM, x = ? (*Required side*)

Then, in rt. $\angle OMP$, we have

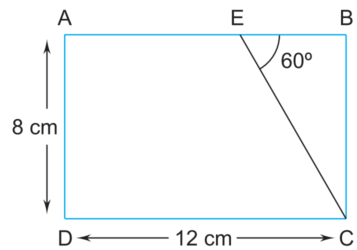
$$\frac{\text{Required side}}{\text{Known/Given side}} = \text{T-Ratio of the given angle.}$$

$$\Rightarrow \quad \frac{OM}{OP} = \text{T-Ratio of the given angle.}$$

$$\Rightarrow \quad \frac{x}{6} = \cos 60^\circ$$

$$\Rightarrow \quad \frac{x}{6} = \frac{1}{2} \quad \Rightarrow \quad x = \frac{6}{2} = 3 \text{ cm.}$$

Example 17: *In the figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE drawn, making an angle of 60° with AB, intersecting AB in E. Find the length of CE.*



Solution: From the figure,

$$BC = AD = 8 \text{ cm}$$

(Opposite sides of a rectangle)

In right ΔEBC, we have

$$\frac{BC}{CE} = \sin 60°$$

$$\left(\frac{\text{Required side}}{\text{Known side}} = \text{T-Ratio of the given angle}\right)$$

$$\Rightarrow \quad \frac{BC}{CE} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad CE = \frac{8}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}}$$

(Rationalizing the denominator)

$$= \frac{16}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{16\sqrt{3}}{3} = 9.24 \text{ cm}$$

EXERCISE 7.5

1. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tide to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
2. From the top of a 50 m high building, the angle of depression of a car on the road is 30°. Find how far the car is from the building ?

3. An observer 1.5 m tall is 18.5 m away from the tower. The angle of elevations of the top of the tower from his eyes is 45°. What is the height of the tower?

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

7.7. USES OF TRIGONOMETRY

It may not have direct applications in solving practical issues but is used in various field. For example, trigonometry is used in developing computer music: as you are familiar that sound travels in the form of waves and this wave pattern, through a sine or cosine function for developing computer music. Here are a few applications where trigonometry and its functions are applicable.

(i) Trigonometry to Measure Height of a Building or a Mountain: Trigonometry is used in measuring the height of a building or a mountain. The distance of a building from the viewpoint and the elevation angle can easily determine the height of a building using the trigonometric functions.

(ii) Trigonometry in Criminology: Trigonometry is even used in the investigation of a crime scene. The functions of trigonometry are helpful to calculate a trajectory of a projectile and estimate the causes of a collision in a car accident. Further, it is used to identify how an object falls or at what angle the gun is shot.

(iii) Trigonometry in Marine Biology: Trigonometry is often used by marine biologists for measurements to figure out the depth of sunlight that affects algae to photosynthesis. Using the trigonometric function and mathematical models, marine biologists estimate the size of larger animals like whales and also understand their behaviours.

(iv) Trigonometry in Navigation: Trigonometry is used in navigating directions; it estimates in what direction to place the compass to get a straight direction. With the help of a compass and trigonometric functions in navigation, it will be easy to pinpoint a location and also to find distance as well to see the horizon.

(v) Other Uses of Trigonometry

- The calculus is based on trigonometry and algebra
- The fundamental trigonometric functions like sine and cosine are used to describe the sound and light waves

- Trigonometry is used in oceanography to calculate the heights of waves and tides in oceans
- It is used in the creation of maps
- It is used in satellite systems

7.8. PROBLEMS INVOLVING ROTATION

So far we have studied angles as parts of triangles, but we can also use angles to describe rotation. For example, rotation of the minute hand on a clock and revolution of a point on a rotating wheel. Every hour, the minute hand moves through one complete rotation or 360°. In two hours, the minute hand rotates through two complete rotations or 720° and so on. Thus, the measure of an angle is the amount of rotation.

Example 18: *Find the angle made by the tip of a minute hand of a watch in 40 minutes.*

Solution: In 60 minutes, the minute hand of a watch completes one rotation *i.e.*, 360°.

In 40 minutes, the minute hand turns through $\frac{40}{60} = \frac{2}{3}$ of a rotation.

Therefore, angle made by the minute hand in 40 minute is given by:

$$\theta = \frac{2}{3} \times 360° = 240°$$

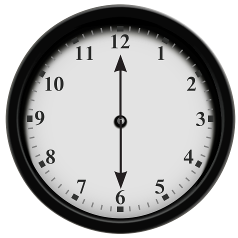
240°

Example 19: *Through how many degrees does the minute hand rotate in an hour and a half ?*

Solution: In 1 hour, the minute hand of a watch completes one rotation i.e., 360°.

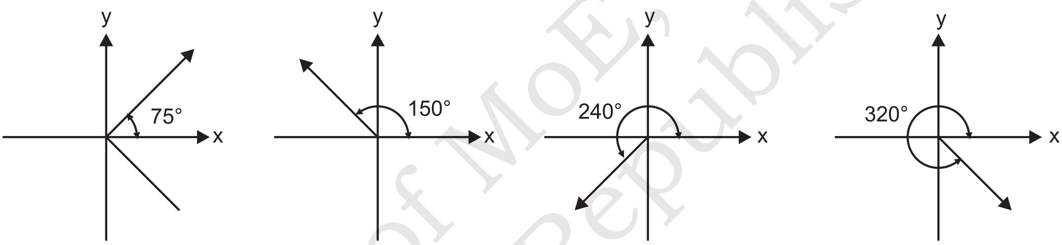
Therefore, in an hour and a half, it completes 1.5 complete rotations. The degrees represented by the minute hand in an hour and a half are given by:

$$\theta = 1.5 \times 360° = 540°$$



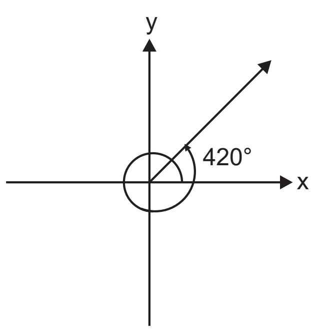
Angles in Standard Position

The degree measure of an angle depends only on the fraction of a whole rotation between its sides, and not on the location or position of the angle. To compare and analyze angles, we place them in **standard position**, so that the vertex of the angle is located at the origin and its initial side lies on the positive *x*-axis, The figure below shows several angles placed in standard position.



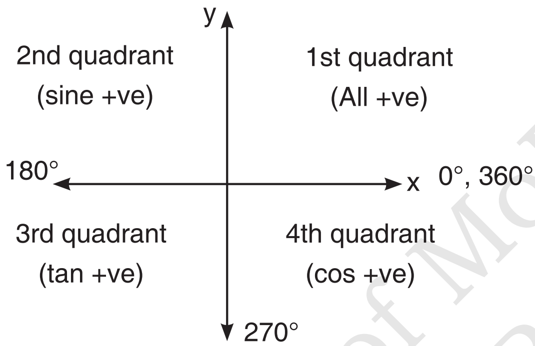
One-quarter of a complete revolution is 90°, one-half of a complete revolution is 180°, and three-quarters of one revolution is 270°. Thus, for angles between 0° and 90° in standard position, the terminal side lies in the 1st quadrant, for angles between 90° and 180° in standard position, the terminal side lies in the 2nd quadrant, for angles between 180° and 270° in standard position, the terminal side lies in the 3rd quadrant, and for angles between 270° and 360°, the terminal side lies in the 4th quadrant.

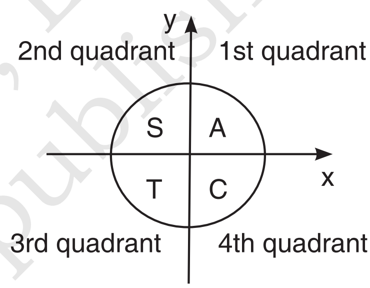
Any angle greater than 360° can be represented in one of the four quadrants, For example, 420° is given in figure.



Trigonometric Ratios of the General Angle

The three main trigonometric ratios may be positive or negative depending on the quadrant in which they lie and their values are related to the corresponding acute angles with the *x*-axis. The signs of the trigonometric ratios can be remembered using the **CAST** diagram as shown below. In the **1st quadrant** (0° to 90°), all the ratios are positive i.e., sine, cosine and tangent give positive values. In the **2nd quadrant** (90° to 180°), only sine gives positive values, cosine and tangent give negative values. In the **3rd quadrant** (180° to 270°), only tangent gives positive values, sine and cosine given negative values. In the **4th quadrant** (270° to 360°), only cosine is positive.





Notes:

1. In trigonometry, positive angles are measured in **anticlockwise direction** from the positive *x*-direction, starting from 0° to 360°.
2. For any angle greater than 360° or multiple of 360°,

$$\sin (n \times 360° + \theta) = \sin \theta$$
$$\cos (n \times 360° + \theta) = \cos \theta$$
$$\tan (n \times 360° + \theta) = \tan \theta$$

Steps to Find the Trigonometric Ratio of Any Angle

Step 1: Sketch the angle in relation to the *x* and *y* axes.

Step 2: Find the 'related/associated acute angle' for the given angle i.e., the acute angle the radius makes with the *x*-axis.

Step 3: Find the appropriate sign of the ratio using the **CAST** diagram. This shows which ratios are positive in each quadrant.

Step 4: Find the required ratio of related angle and attach the appropriate sign.

Note: For trigonometric ratio of angles beyond 90°, we can find the related or associated angle as:

- If θ lies in the 2nd quadrant, the acute angle $(180° - \theta)$ is the related angle for θ.
- If θ lies in the 3rd quadrant, the acute angle $(\theta - 180°)$ is the related angle for θ.
- If θ lies in the 4th quadrant, the acute angle $(360° - \theta)$ is the related angle for θ.
- If θ is greater than 360°, the related angle will be find after sketching the angle in one of the four quadrant and applying the above mentioned method.

Example 20: *Find the values of*

(*i*) *sin 150°* (*ii*) *cos 210°*

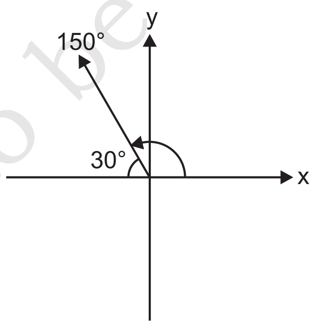
(*iii*) *tan 315°* (*iv*) *sin 420°*

Use a calculator to verify your answers.

Solution:

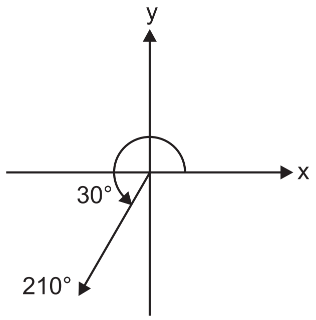
(i) 150° lies in the 2nd quadrant, therefore its sine ratio is positive. The related acute angle with the *x*-axis is 30°

$$\therefore \quad \sin 150° = +\sin 30° = \frac{1}{2} = 0.5$$



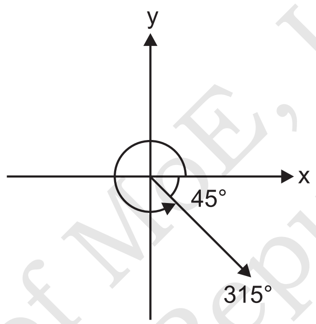
(ii) 210° lies in the 3rd quadrant, therefore its cosine ratio is negative. The related acute angle with the *x*-axis is 30°

$$\therefore \quad \cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2} = -0.866$$



(iii) 315° lies in the 4th quadrant, therefore its tangent ratio is negative. The related acute angle with the *x*-axis is 45°.

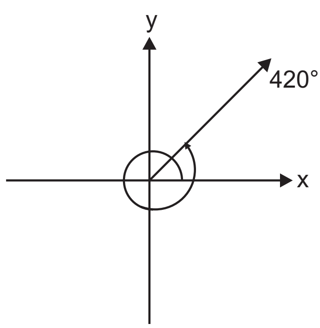
$$\therefore \quad \tan 315^\circ = -\tan 45^\circ = -1$$



(iv) 420° lies in the first quadrant, therefore its sine ratio is positive. The angle is greater than 360° and it makes one rotation and comes in the 1st quadrant at 60°.

$$\sin (360^\circ + 60^\circ) = \sin 60^\circ$$

$$\therefore \quad \sin 420^\circ = \frac{\sqrt{3}}{2} = 0.866$$



EXERCISE 7.6

1. Find the angles made by the tip of a minute hand of a watch in
   (i) 20 minutes (ii) 30 minutes
   (iii) 45 minutes (iv) 2 hours and a half
2. A wheel is rotating at 300 revolutions per minute. How many degrees does a point on the edge move in
   (i) 1 second (ii) 5 seconds
   (iii) $8\frac{3}{5}$ seconds
3. What is the sign of
   (i) $\cos 150°$ (ii) $\sin 300°$
   (iii) $\tan 235°$ (iv) $\sin 800°$
4. Use the related/associated angle to find exact value of.
   (i) $\cos 240°$ (ii) $\sin 120°$
   (iii) $\tan 300°$ (iv) $\sin 210°$
   (v) $\tan 570°$ (vi) $\cos 330°$
   (vi) $\tan 150°$ (viii) $\cos 750°$

MULTIPLE CHOICE QUESTIONS

1. If Δ ABC is right angled at B, then which of the following is true?
   (a) $\frac{AB}{AC} = \sin C$ (b) $\frac{BC}{AB} = \tan C$
   (c) $\frac{AC}{AB} = \cos C$ (d) $\frac{AB}{AC} = \sec C$
2. The value of $\frac{2\tan 30°}{1+\tan^2 30°}$ is
   (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{2}}$
   (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

3. The value of $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 30^\circ}$ is

(a) $\tan 90^\circ$ (b) 1

(c) $\sin 45^\circ$ (d) 0

4. The value of $(\sin 45^\circ + \cos 45^\circ)$ is

(a) 1 (b) $\dfrac{1}{\sqrt{2}}$

(c) $\dfrac{\sqrt{3}}{2}$ (d) $\sqrt{2}$

5. The value of $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$ is

(a) -1 (b) 0

(c) 1 (d) 2

6. The value of the expression $\left[\dfrac{\cos 60^\circ + \sin 60^\circ}{\cos 60^\circ - \sin 60^\circ}\right]$ is

(a) $-(\sqrt{3}+2)$ (b) -1

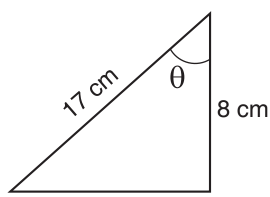
(c) $(\sqrt{3}-2)$ (d) $(\sqrt{3}+2)$

7. If $\cos\theta = \dfrac{4}{5}$, then the value of $\sin\theta$ is

(a) $\dfrac{3}{5}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{3}{4}$ (d) $\dfrac{1}{5}$

8. In the figure, the value of θ is



(a) 53.1° (b) 61.9°

(c) 60.1° (d) None of these

9. At an instant, the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole. Then, the angle of elevation of the sun is

(a) 30° (b) 45°

(c) 60° (d) 75°

10. A kite is flying at a height of 75 m from the level ground, attached to a string inclined at 60° to the horizontal. Then, the length of the string is

(a) $50\sqrt{2}$ m (b) $50\sqrt{3}$ m

(c) $\frac{50}{\sqrt{2}}$ m (d) $\frac{50}{\sqrt{3}}$ m

11. The angle of elevation from the top of a tower from a distance of 100 m from its foot is 30°. Then, the height of the tower is

(a) $100\sqrt{3}$ (b) $\frac{100}{\sqrt{3}}$ m

(c) $50\sqrt{3}$ m (d) $\frac{200}{\sqrt{3}}$ m

12. The angle of elevation of the top of a tower from a point on ground, which is 30 m away from the foot of tower is 45°. Then, the height of tower (in metre) is

(a) 15 (b) 30

(c) $30\sqrt{3}$ (d) $10\sqrt{3}$

13. From a point on the ground, 20 m away from the root of a tree, the angle of elevation of the top of the tree is 45°. Then, the height of the tree is

(a) 20 m (b) 25 m

(c) 15 m (d) None of these

14. If the elevation of the Sun changes from 30° and 60°, then the difference between the length of shadows of a pole 15 m high at these two elevations of the Sun, is

(a) 7.5 m (b) 15 m

(c) $10\sqrt{3}$ m (d) $\frac{15}{\sqrt{3}}$ m

15. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is

(a) 60° (b) 45°

(c) 30° (d) 90°

16. A vertical stick 20 m long casts a shadow 16 m long. At the same time a tower casts a shadow 48 m long. Then, the height of the tower is

(a) 32 m (b) 40 m

(c) 60 m (d) 96 m

17. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is

(a) 45° (b) 30°

(c) 60° (d) 90°

18. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in metres) is

(a) $\frac{4}{\sqrt{3}}$ (b) $4\sqrt{3}$

(c) $2\sqrt{2}$ (d) 4